A Novel Approach to the Analysis of Microwave Regenerative Analog Frequency Dividers

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***Abstract*—This letter presents a CAD-oriented approach to the analysis of regenerative analog frequency dividers based upon the Volterra series method. Design information such as locking frequency range and minimum injected power levels can easily be found from the determining equation thus obtained. Locking performances of a MESFET analog 1/3 frequency divider evaluated by the proposed method and a time-domain simulator are included.**

***Index Terms*—Regenerative frequency dividers, Volterra series.**

I. Introduction

APPLICATIONS of frequency- and phase-locked components can be found in many microwave systems [1] such as phase-locked loops or adaptive phase array. These components include injection-locked oscillators and frequency dividers. A number of reports have been published on the analysis of injection-locked oscillators including time-domain methods [2], frequency-domain methods [3], and harmonic balance algorithms [4]. In this letter, a simple, fast, and general approach to the analysis of regenerative analog frequency dividers based upon the Volterra series method is presented. The novel feature of the proposed method here is the way in which the divider circuit is decomposed and analyzed. Unlike time-domain technique and harmonic balance algorithms, both long computation time and problem of convergence are avoided. Its application to the analysis of a MESFET analog 1/3 frequency divider is demonstrated.

II. Formulation of Determining Equation

Regenerative analog $1/N$ frequency dividers can generally be modeled by the circuit shown in Fig. 1(a), which consists of a linear (Z_L) and a nonlinear blocks (Z_{NL}), driven by an external current source with amplitude I_j and angular frequency $N\omega_j$. In the absence of any injected signal $(I_j = 0)$, the circuit is simply a free-running oscillator. For derivation purposes, the two-element oscillator is transformed into the equivalent circuit shown in Fig. 1(b) where $Z_s(\omega)$ and $Z_p(\omega)$ are bandstop and bandpass filters defined by

$$Z_s(\omega) = \begin{cases} Z_L(\omega), & \text{otherwise} \\ \infty, & \omega = \omega_j \end{cases} \tag{1}$$

$$Z_p(\omega) = \begin{cases} Z_L(\omega), & \omega = \omega_j \\ \infty, & \text{otherwise.} \end{cases} \tag{2}$$

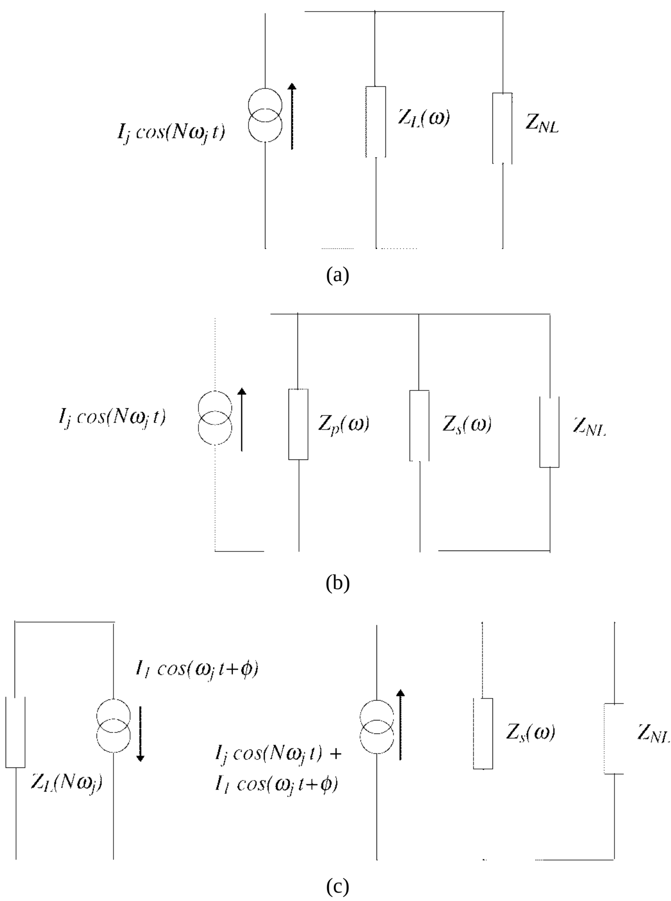

Fig. 1. Regenerative frequency divider. (a) General configuration. (b) Equivalent model. (c) Open-loop model.

Under the locking condition, it is possible to decompose the oscillator into two subcircuits as depicted in Fig. 1(c), where I_1 and ϕ are, respectively, the amplitude and phase of the fundamental component (ω_j) of the branch current flowing through Z_L. Since the voltages across the two subcircuits are identical, hence we have

$$Z_L(\omega_j)\frac{I_1}{2}e^{j\omega_j t+\phi} + V_{NL}(\omega_j) = 0 \tag{3}$$

where $V_{NL}(\omega_j)$ is the fundamental component of the voltage across the input of the nonlinear subcircuit driven by the two-tone current excitation defined previously. This voltage can be computed very efficiently by the nonlinear current method described in [5]. Upon examining the determining equation in (3), it is obvious that $V_{NL}(\omega_j)$ is a function of only four variables, namely, ω_j, I_1, I_j, and ϕ. Therefore, the

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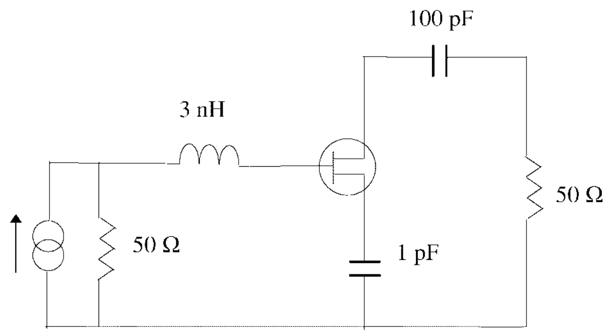

Fig. 2. MESFET analog 1/3 frequency divider circuit.

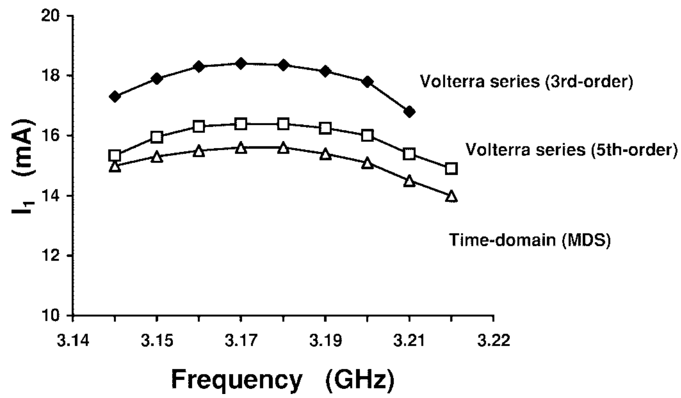

Fig. 3. Plot of current I_1 versus locking frequency ($I_j = 18$ mA).

frequency divider problem has been reduced to the solving of two nonlinear algebraic equations (real and imaginary parts of the determining equation) in two unknowns (I_1 and ϕ) with prespecified ω_j and I_j values. The set of curves thus obtained (plot of I_j versus I_1 with ω_j as parameter) is useful for the determination of design information such as the locking frequency range of the divider circuits.

III. Application to MESFET Analog 1/3 Frequency Divider

Based upon Volterra series analysis, the determining equation for an analog 1/3 frequency divider may be derived as

$$\begin{aligned} &Z_L(\omega_j)\frac{I_1}{2} + H_1(\omega_j)\frac{I_1}{2} + H_3(\omega_j,\omega_j,-\omega_j)\frac{3I_1^3}{8} \\ &\quad + H_3(3\omega_j,-3\omega_j,\omega_j)\frac{6I_1I_j^2}{8} \\ &\quad + H_3(3\omega_j,-\omega_j,-\omega_j)\frac{3I_1^2}{8}I_je^{-j3\phi} + \cdots = 0 \qquad (4) \end{aligned}$$

where $H_n(\ )$ is the Volterra kernel of nth order [5]. Note that $H_1(\omega)$ is simply the input impedance of the nonlinear subcircuit in the absence of nonlinearities. With low injected current assumption, an extremely simple expression for I_j may be obtained from (4), by neglecting all Volterra kernels higher than the third order:

$$I_j = \frac{\left|Z_L(\omega_j) + H_1(\omega_j) + H_3(\omega_j,\omega_j,-\omega_j)\dfrac{3I_1^2}{4}\right|}{\dfrac{3I_1}{4}\left|H_3(3\omega_j,-\omega_j,-\omega_j)\right|}. \qquad (5)$$

If one is interested in a "nearly" exact solution of (4), one could always resort to a more efficient root-finding algorithm using the above "approximate" values as an initial estimates.

In order to study the accuracy of the proposed method, the locking performances of a MESFET analog 1/3 frequency divider (Fig. 2) has been analyzed. The equivalent circuit model of the transistor used in these computations is obtained from [3]. By following the procedure described in Section II, both third- and a fifth-order solutions for current I_1 (versus locking frequency) are calculated and shown in Fig. 3. The free-running oscillating frequency of the circuit is found to be 3.18 GHz. For comparison purposes, a time-domain simulation of the same circuit is also performed using a commercial CAD tool (HP MDS). The figure shows that there is an excellent agreement between the two different approaches, even for the approximate third-order solution.

IV. Conclusions

A new Volterra series approach for the analysis of analog frequency dividers has been described. It has been demonstrated that the proposed method is highly accurate on the prediction of the locking performances of the frequency divider circuit under various injected current levels. The application of the proposed method to a MESFET frequency divider is demonstrated and a simple and accurate third-order solution is also suggested.

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